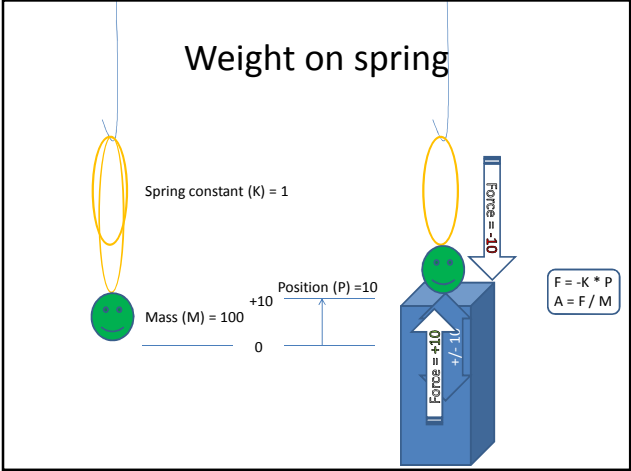


Resonance made simple

(really)



Computing changes in position

- From previous slide:
 - $F = -K * P$
 - $A = F / M$
- $A = -K * P / M$
- Speed change in 1s
 - $V = V + A$
- Position change 1s
 - $P = P + V$

Constants

K: spring constant (N/m)

M: mass (kg)

Variables

P: position (m)

F: force (N)

A: acceleration (m/s)

V: velocity (m/s²)

```
PROGRAM: RESONATE
:10→P:0→V
:1→K:100→M
:For(T,0,92)
:Pt-On(T,P)
:V+ -P*K/M→V
:P+V→P:End
```

Relating position and velocity

```
PROGRAM: RESONATE
:10→P:0→V
:1→K:100→M
:For(T,0,92)
:Pt-On(T,P)
:V+ -P*K/M→V
:P+V→P:End
```

```
PROGRAM: RESONATE
:10→P:0→V
:1→K:100→M
:For(T,0,92)
:Pt-On(T,P)
:V+ -P*K/M→V
:P+V→P:End
```

Period (time to repeat)

Stationary when P extreme

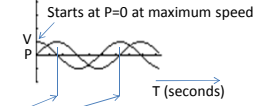
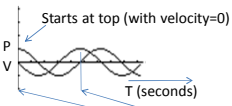
Kinetic energy (when fastest) = $\frac{1}{2}mv^2 = \frac{1}{2} * 100 * 1 = 50$

Potential energy in spring (when stopped) = $\frac{1}{2}kp^2 = \frac{1}{2} * 1 * 100 = 50$

Relating position and velocity

```
PROGRAM: RESONATE
:0→P:0→V
:1→K:100→M
:For(T,0,92)
:Pt-On(T,P)
:Pt-On(T,V*10)
:V+P*K/M→V
:P+V→P:End
```

```
PROGRAM: RESONATE
:0→P:1→V
:1→K:100→M
:For(T,0,92)
:Pt-On(T,P)
:Pt-On(T,V*10)
:V+P*K/M→V
:P+V→P:End
```

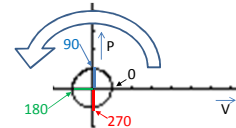
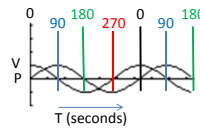


Equivalent "phases"

Plotting Velocity v. Position

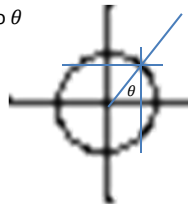
```
PROGRAM: RESONATE
:0→P:1→V
:1→K:100→M
:For(T,0,92)
:Pt-On(T,P)
:Pt-On(T,V*10)
:V+P*K/M→V
:P+V→P:End
```

```
PROGRAM: RESONATE
:0→P:1→V
:1→K:100→M
:For(T,0,92)
:Pt-On(T,P)
:Pt-On(T,V*10)
:V+P*K/M→V
:P+V→P:End
```



So, what do we make of this

- Sine represents a simple periodic process
 - Most convenient representation is angle
- Correspond to locations on a unit circle
 - θ is an angle
 - $\cos \theta$ is the x coordinate corresponding to θ
 - $\sin \theta$ is the y coordinate corresponding to θ
- Due to Pythagorean theorem
 - $\sin^2 \theta + \cos^2 \theta = 1$
- An angle θ can either be measured
 - In degrees (0...360 or -180...180)
 - In circumference (0... 2π or $-\pi \dots \pi$)



Exercises (& Homework)

- For each of the following, describe what happens and explain why
 1. If initially V is 0.5 or 2?
 2. If initially P=5 and V=1
 3. If M is doubled?
 4. If K is doubled
 5. If K & M are doubled?